Lesson 1

Scattering, Diffraction, and Radiation

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Various slides under courtesy of Prof. R. Trebino at GIT

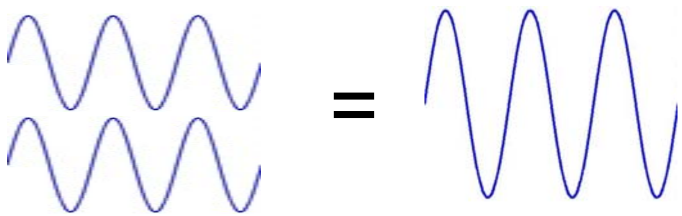
Outline

- Scattering
- Diffraction
  - Grating as example
  - Requires coherence
- Radiation: where does light come from
  - Revisit Brewster's angle

Coherent interference

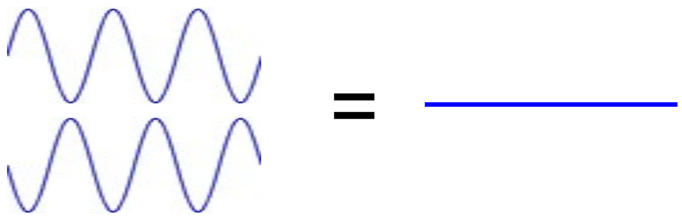
Again, a result of **coherence**: phase stability is crucial!!!

Waves that combine
in phase add up to
relatively high irradiance.



Constructive
interference

Waves that combine **180°**
out of phase cancel out
and yield zero irradiance.



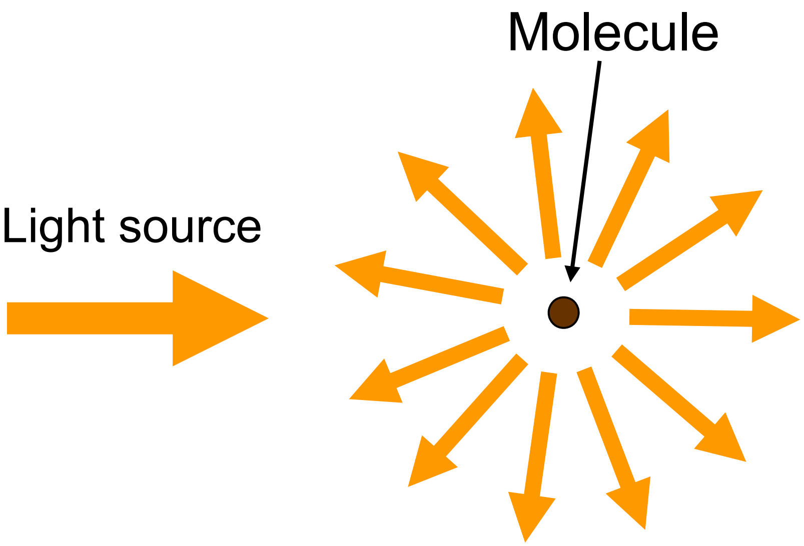
Destructive
interference

- Scattering
  - Spherical wavelets
  - Colors
  - Scattering + Interference → Law of reflection

Light scattering

When light encounters matter, matter re-emits light in all other directions.

This is called **scattering**



Light scattering is everywhere. All molecules scatter light. Surfaces scatter light. Scattering causes milk and clouds to be white. It is the basis of nearly all optical phenomena.

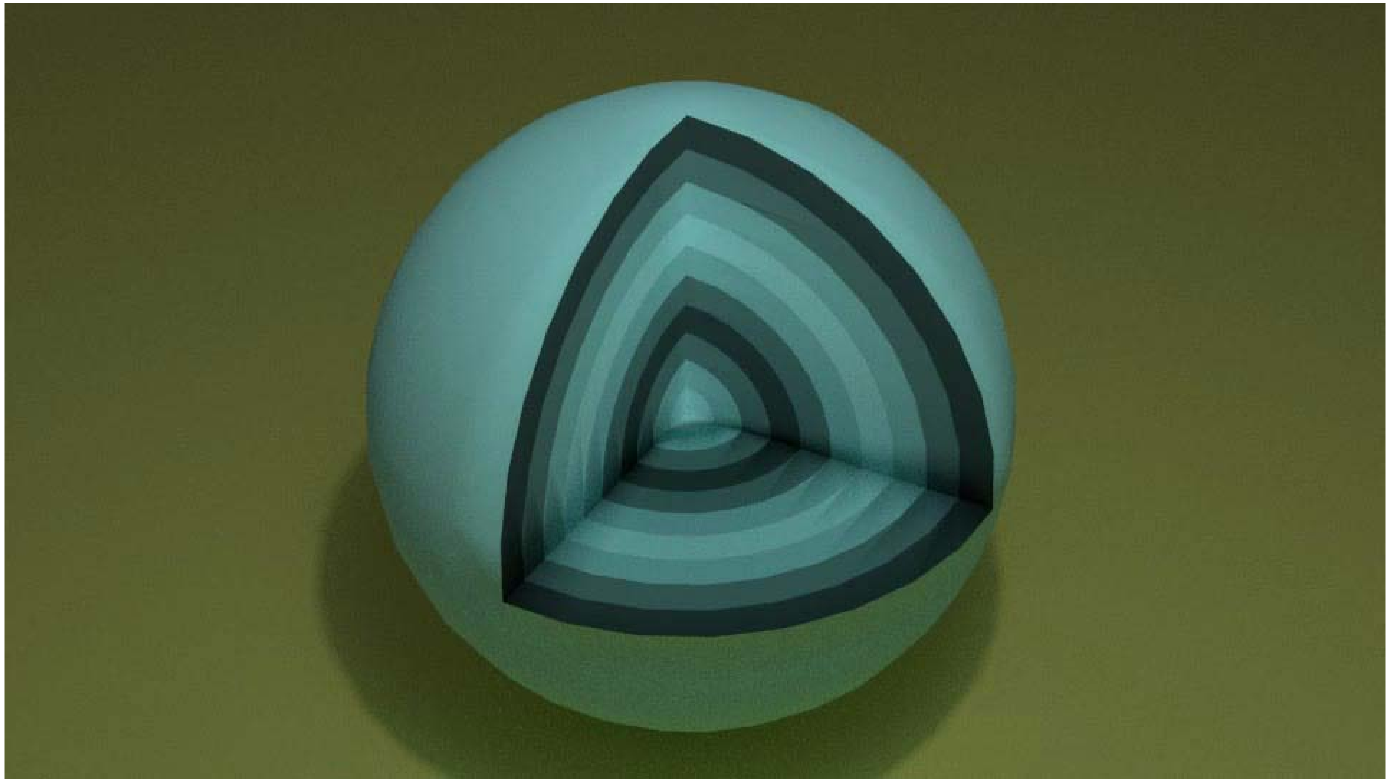
Let's do an experiment!!

Spherical waves

A spherical wave is also a solution to Maxwell's equations and is a good model for the light scattered by a molecule.

$$E(\vec{r},t) = \frac{E_0}{r}\mathrm{Re}\{\exp[\,j(\omega t - kr)]\}$$

- where k is a scalar, and
- r is the radial magnitude.



Colors due to scattering

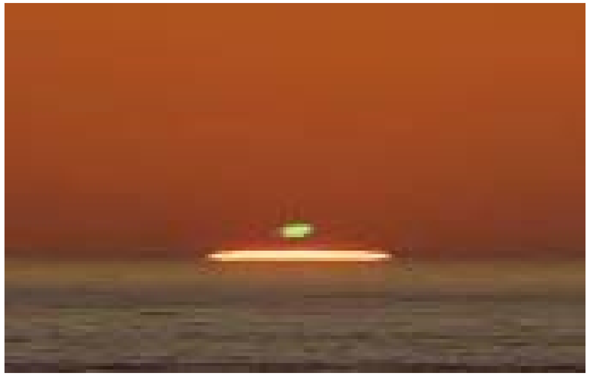
- Blue sky, yellow sunset
- **Rayleigh scattering**

$$S \propto f^4$$

Sunset can be Green!

Just as the sun sets, there is a green flash for a fraction of a second.

Pirates of the Caribbean: At World's End

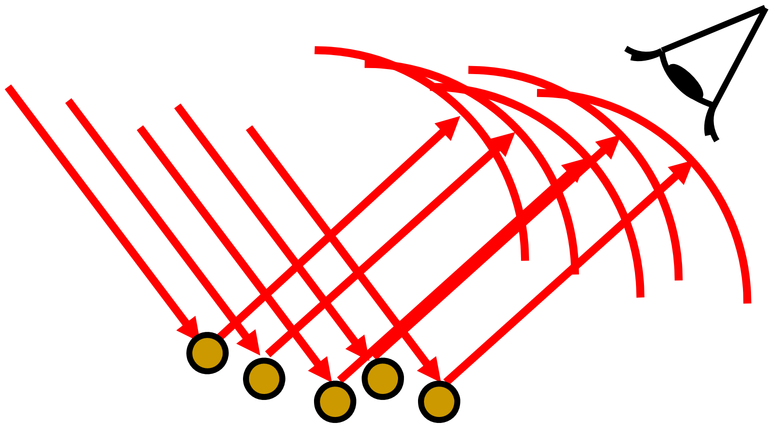


Refractive ray curvature, but the long path allows an otherwise unseen atmospheric absorption in the yellow to split the sun's spectrum into orange and green components.

Check one direction at a time, far away

This way we can approximate spherical waves as plane waves in that direction, vastly simplifying the math.

$$E(\vec{r},t) = \frac{E_0}{r}\mathrm{Re}\{\exp[j(\omega t - kr)]\} \quad \boxed{\Rightarrow E_{r0}\,\mathrm{Re}\{\exp[j(\omega t - kz)]\}}$$



Far away, spherical wave-fronts are almost flat…

Usually, coherent constructive interference will occur in one direction, and destructive interference will occur in all others.

If incoherent interference occurs, it is usually omni-directional.

The mathematics of scattering

The math of light scattering is analogous to that of interference.

If the phases are not random (coherent process), we add the fields:

$$\boxed{\bullet\ E_{total} = E_1 + E_2 + \ldots + E_n}$$

$$I_{total} = \underbrace{I_1 + I_2 + \ldots + I_N}_{} + \underbrace{c\varepsilon \operatorname{Re}\left\{E_1 E_2^* + E_1 E_3^* + \ldots + E_{N-1} E_N^*\right\}}_{}$$

I_1, I_2, … I_n are the irradiances of the various beamlets. They're all positive real numbers and add.

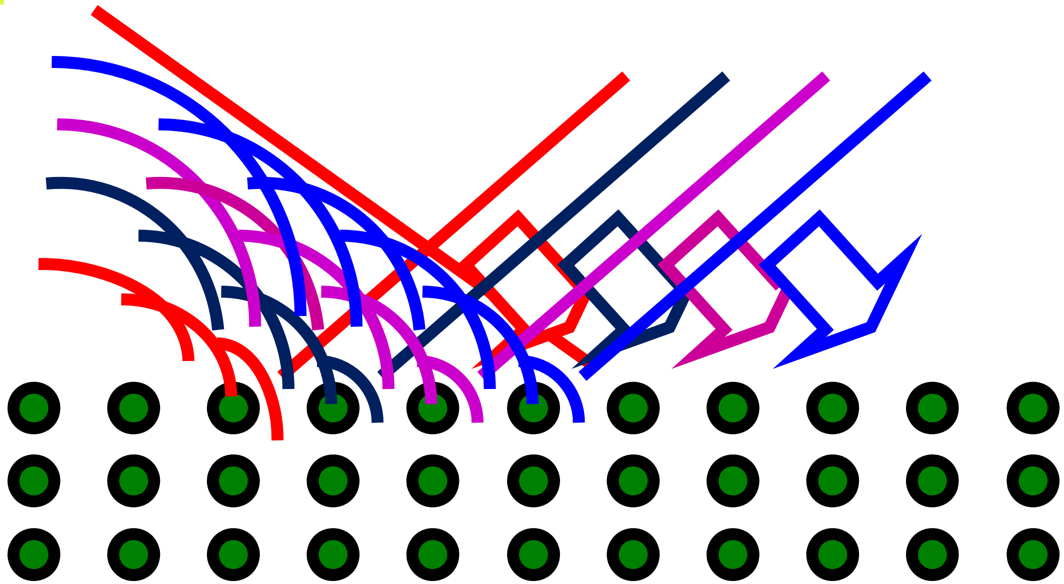
$E_i\, E_j^*$ are cross terms, which have the phase factors: $\exp[j(\theta_i - \theta_j)]$. When the θ's are not random, they don't cancel out!

Scattered spherical waves ➔ plane waves

A plane wave impinging on a surface (that is, lots of very small closely spaced scatterers!) will produce a reflected plane wave because all the spherical wavelets interfere constructively along a flat surface.

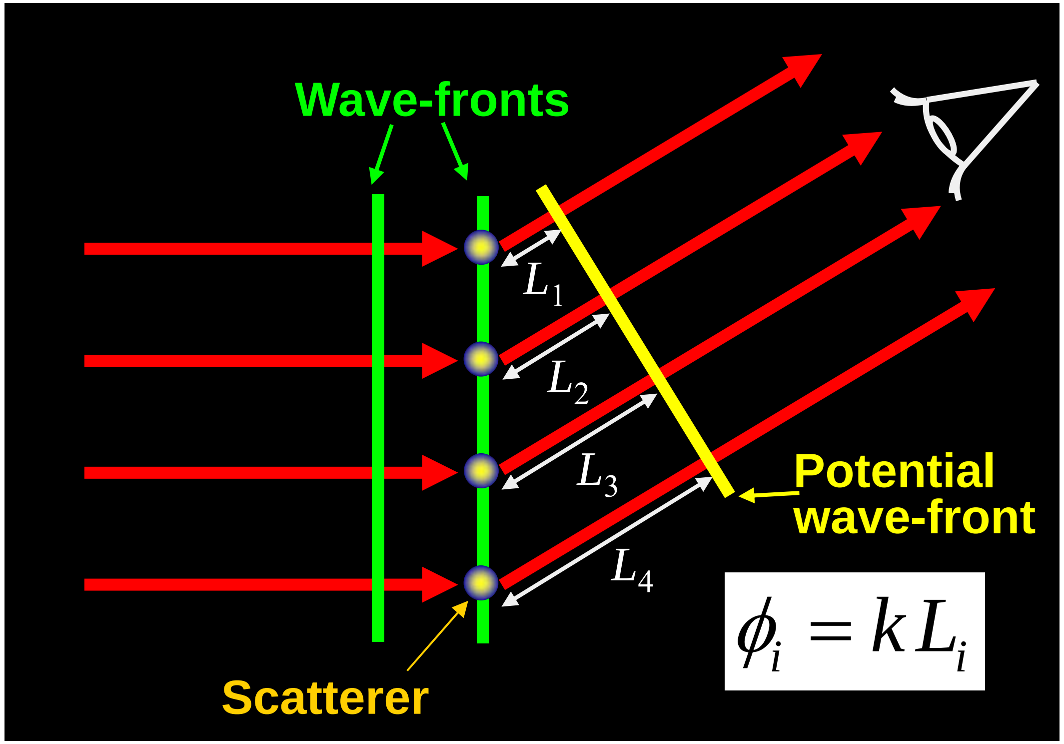
What Huygens taught us

Christian Huygens (1629-1695)



Phase delays

Because the phase is constant along a wave-front, we compute the phase delay from one wave-front to another potential wave-front.



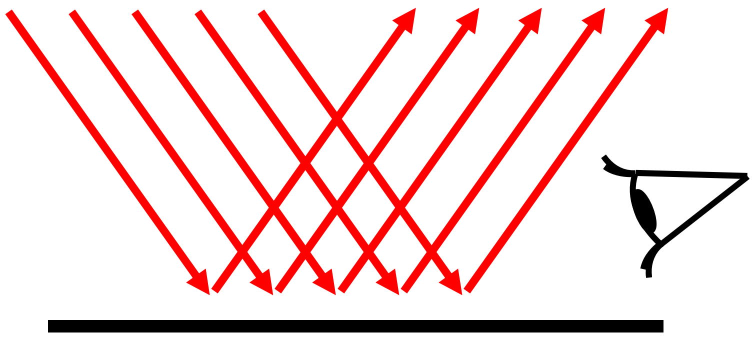
Phase delays all same (modulo 2π): constructive and coherent.

Phases vary uniformly: destructive and coherent.

Phases random: incoherent.

Direction of coherent scattering

A smooth surface scatters light coherently and constructively only in the direction whose **angle of reflection equals the angle of incidence**.

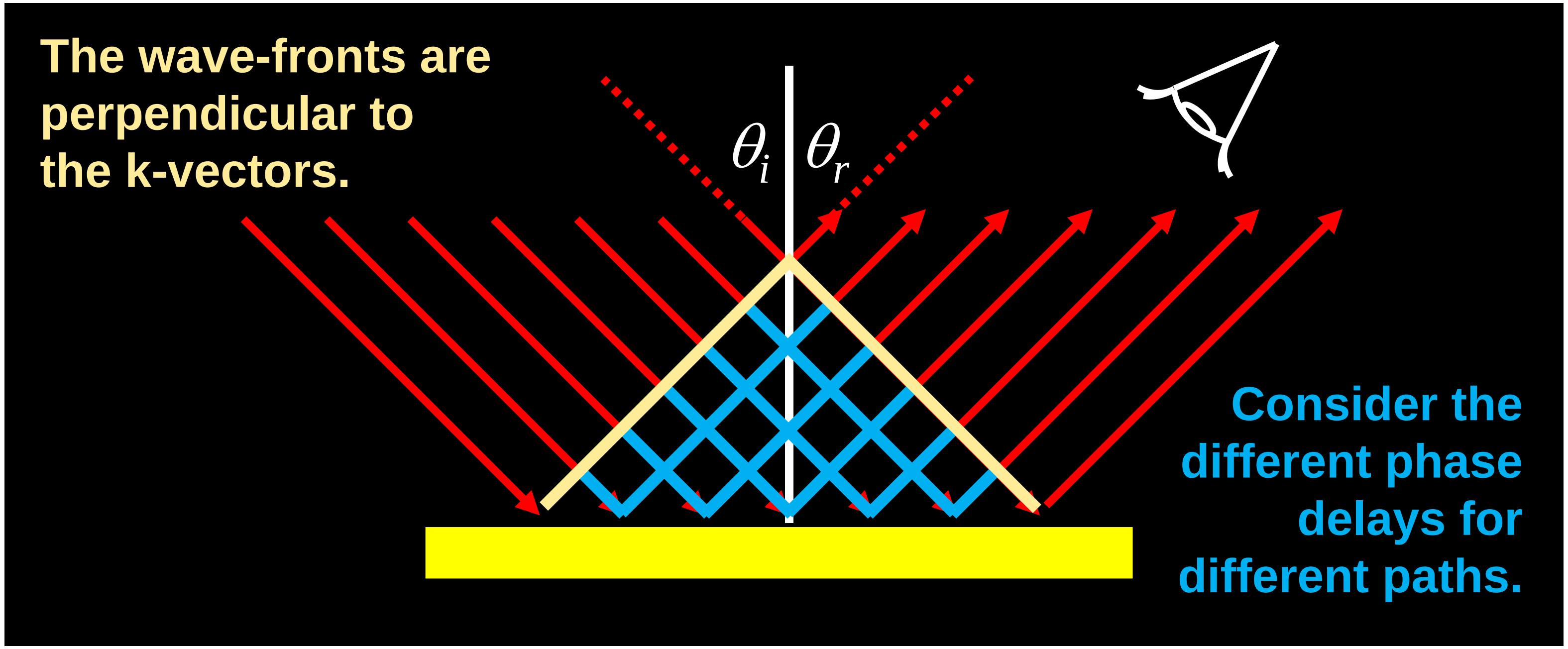


Looking from any other direction, you'll see no light at all due to coherent destructive interference.

Let's do an experiment!!

Coherent constructive scattering

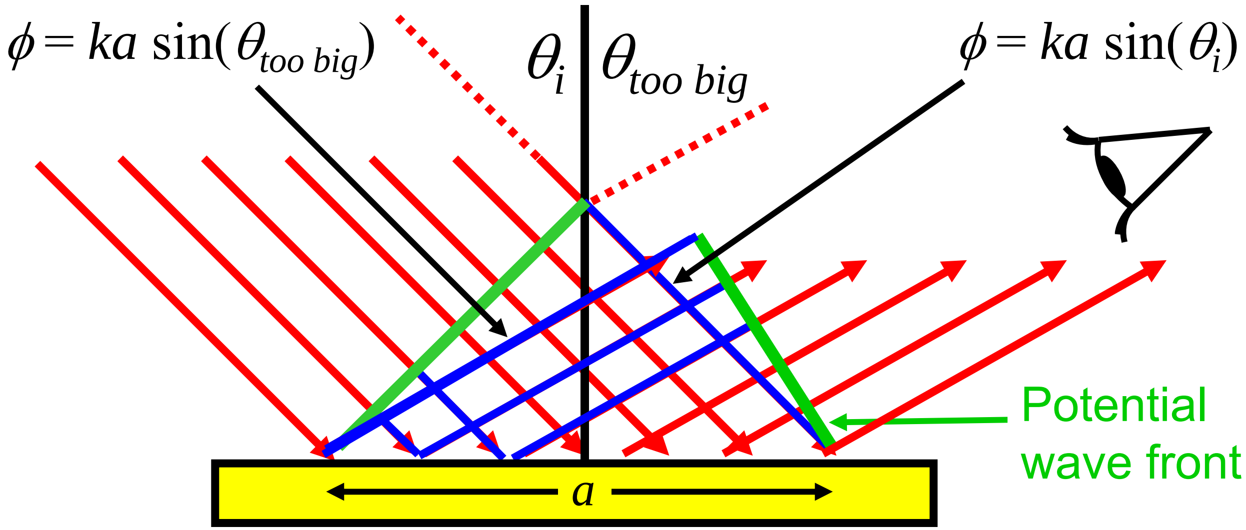
A beam can only remain a plane wave if there's a direction for which coherent constructive interference occurs.



Coherent constructive interference occurs for a reflected beam if the angle of incidence = the angle of reflection: $\theta_i = \theta_r$.

Coherent destructive scattering: different angles

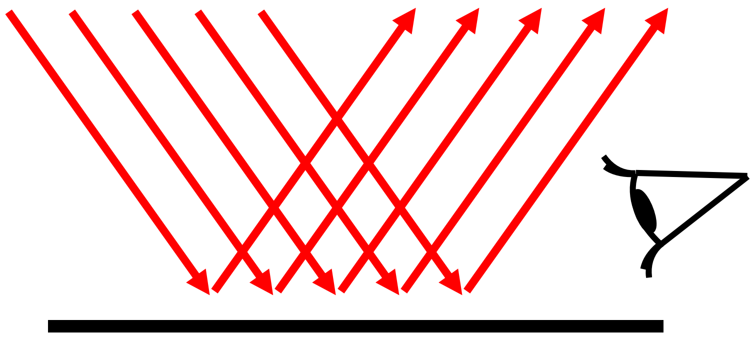
Imagine that the reflection angle is too big, the symmetry is now gone, and the phases (path lengths) are now all different



Coherent destructive interference occurs for a reflected beam direction if the angle of incidence ≠ the angle of reflection: $\theta_i \neq \theta_r$.

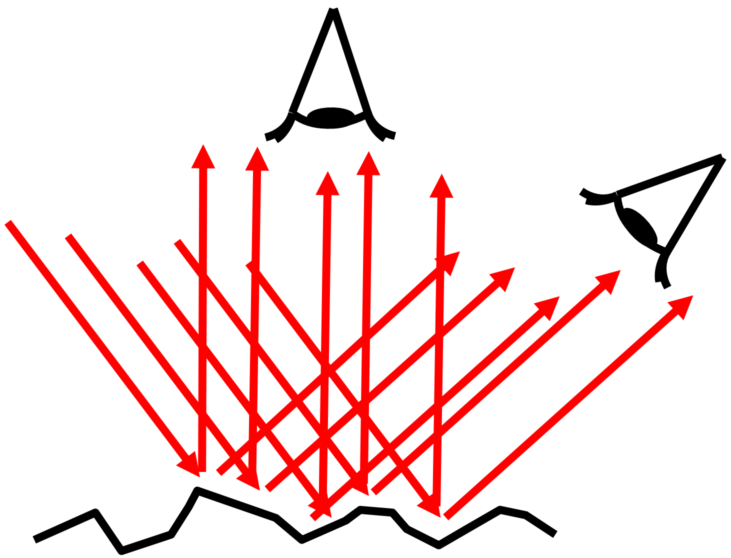
Direction of coherent scattering

A **smooth surface** scatters light coherently and constructively only in the direction whose angle of reflection equals the angle of incidence.



Looking from any other direction, you'll see no light at all due to coherent destructive interference.

Incoherent scattering: rough surface



No matter which direction we look at it, each scattered wave from a rough surface has a different phase. So scattering is incoherent, and we'll see weak light in all directions.

This is why rough surfaces look different from smooth surfaces and mirrors.

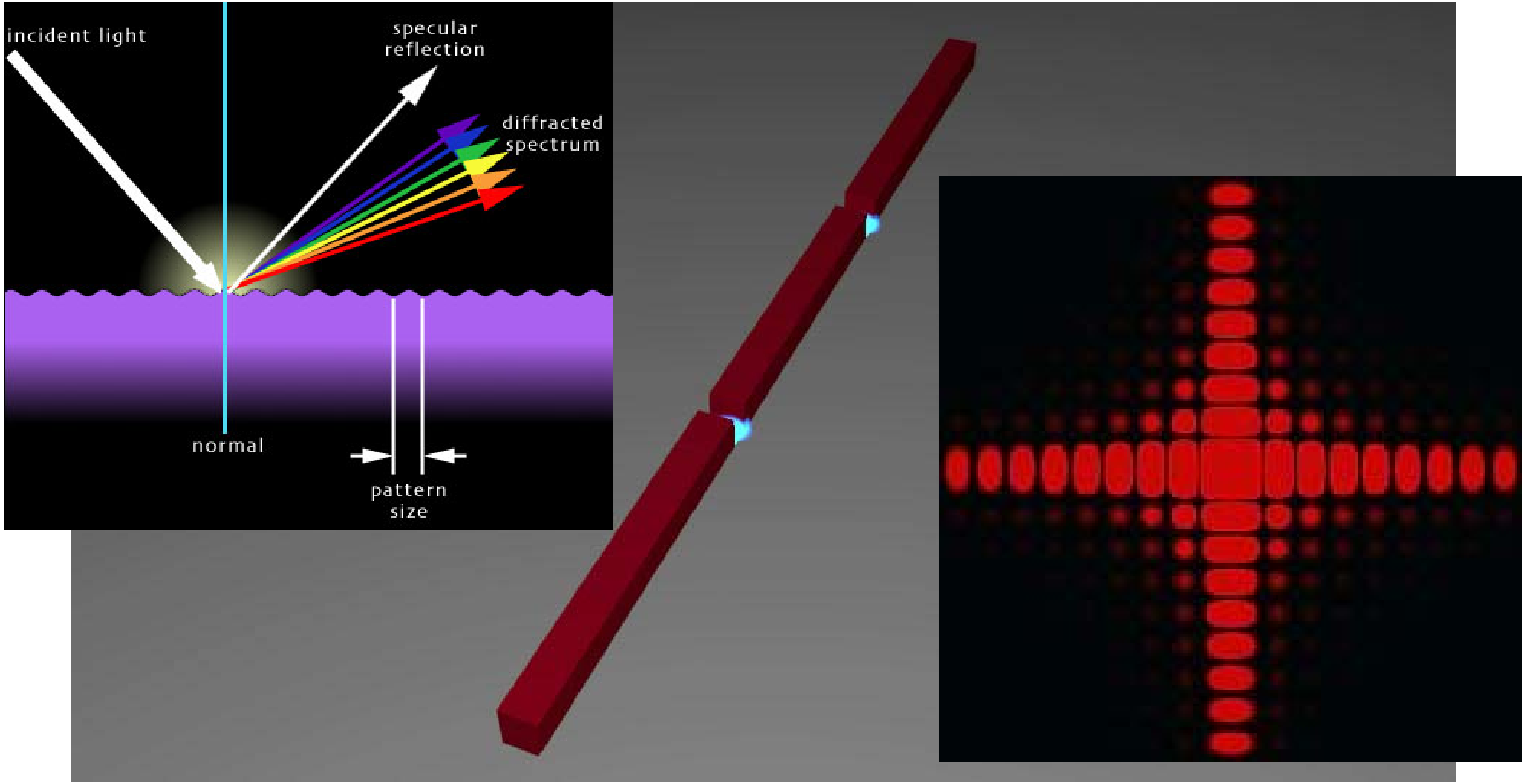
Let's do an experiment!!

- Diffraction
  - Optical grating

Diffraction?

- A coherent process

Scattering **(periodic)** ➔ Interference ➔ Diffraction

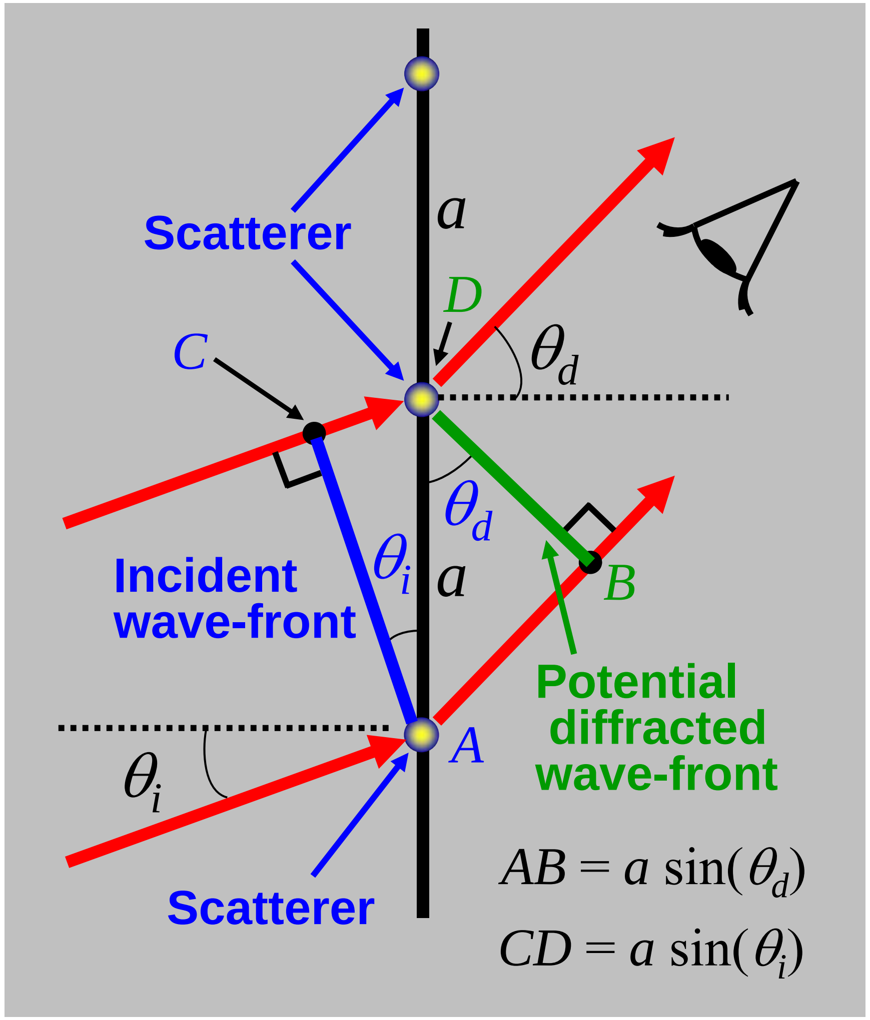


Diffraction gratings

Scattering explain what happens when light impinges on a periodic array of grooves. Constructive interference occurs if the delay between adjacent beamlets is an integral number, *m*, of wavelengths.

Path difference: $AB - CD = m\lambda$

$$a\left[\sin(\theta_d) - \sin(\theta_i)\right] = m\lambda$$



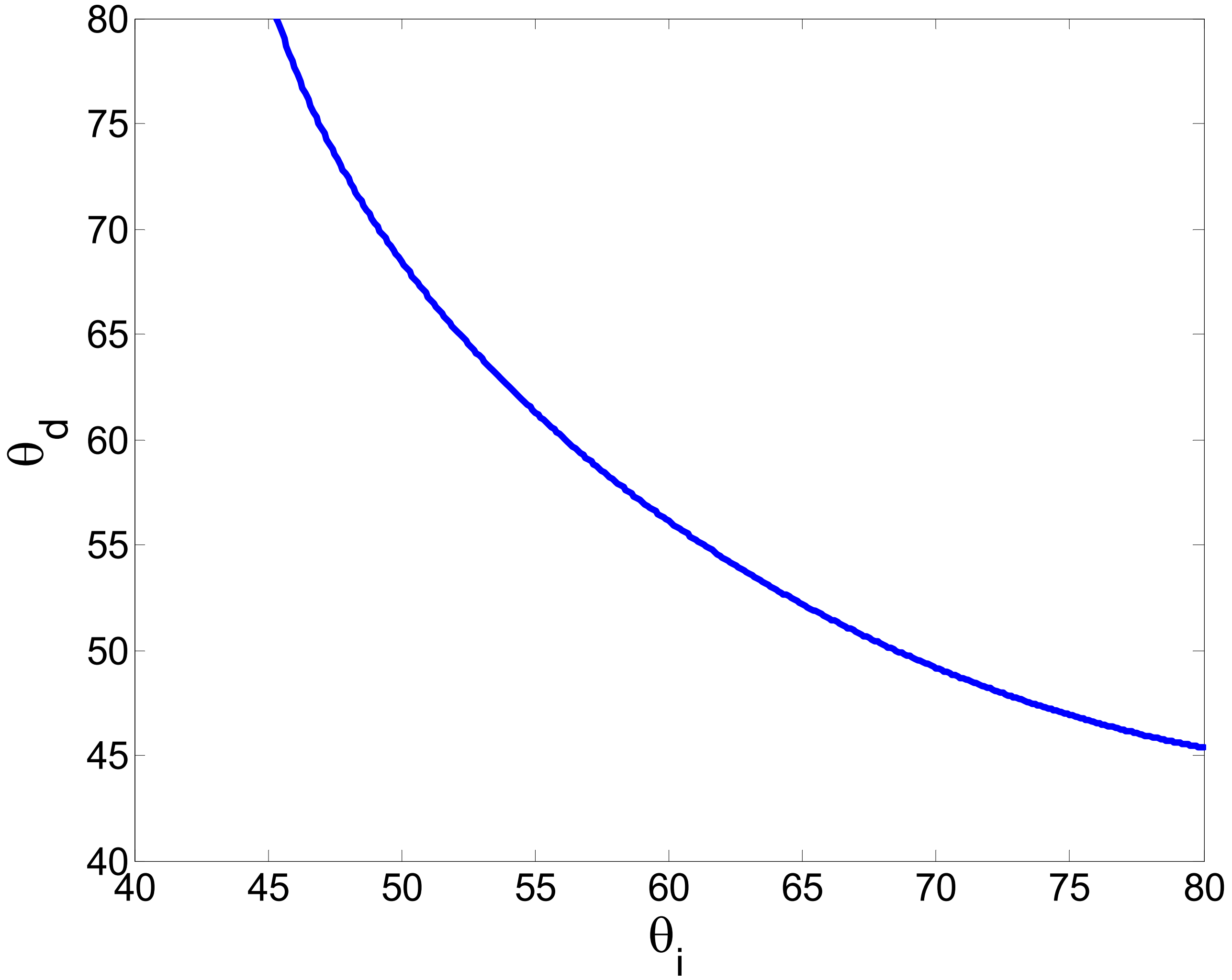
where m is any integer.

A grating has solutions or zero, one, or many values of m, or **orders**.

Remember that m and θ_m can be negative, too.

Incident angle vs. diffraction angle

$$a\left[\sin(\theta_d) - \sin(\theta_i)\right] = \lambda$$



Diffraction-grating dispersion

Because diffraction gratings are used to separate colors, it's helpful to know the variation of the diffracted angle vs. wavelength.

$$a\left[\sin(\theta_d) - \sin(\theta_i)\right] = m\lambda$$

[θ_i is constant]

Differentiating the grating equation, with respect to wavelength:

$$a\cos(\theta_d)\frac{d\theta_d}{d\lambda} = m$$

Rearranging:

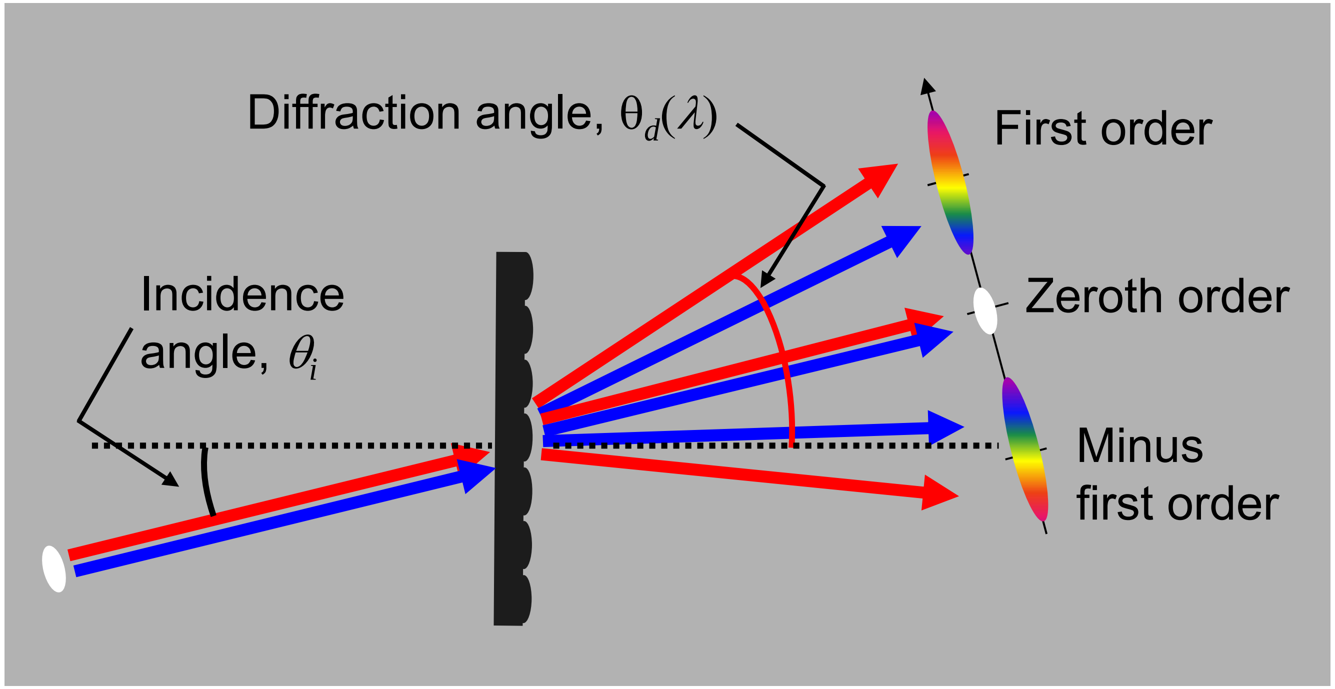
$$\boxed{\frac{d\theta_d}{d\lambda} = \frac{m}{a\cos(\theta_d)}}$$

Gratings typically have an order of magnitude more dispersion than prisms.

Thus, to separate different colors maximally, make a small, work in high order (make m large), and use a diffraction angle near 90 degrees.

Diffraction orders

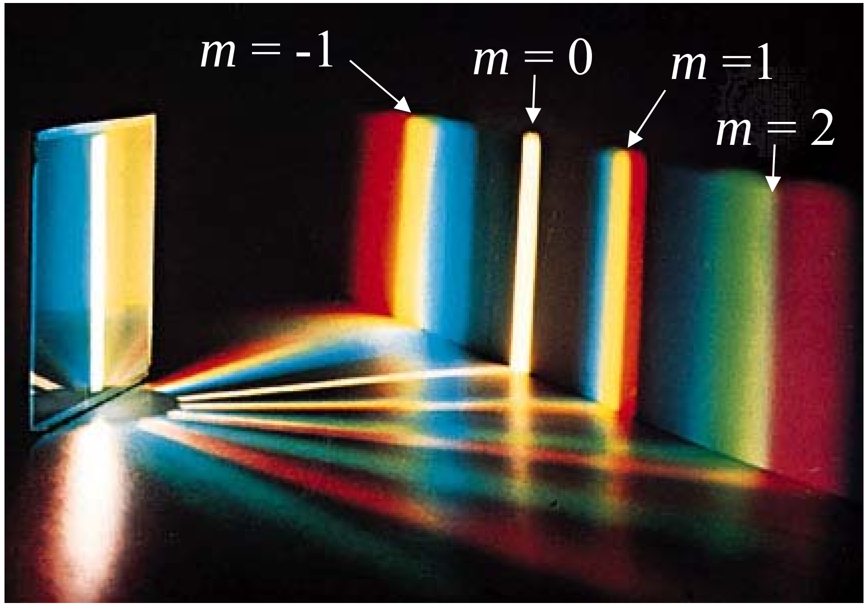
Because the diffraction angle depends on λ, different wavelengths are separated in the nonzero orders.



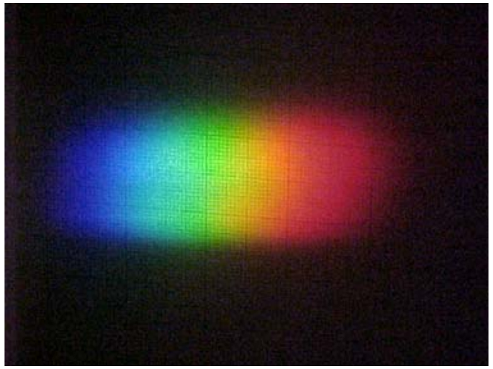
No wavelength dependence occurs in zero order.

The longer the wavelength, the larger its deflection in each nonzero order.

Real diffraction gratings



Diffracted white light

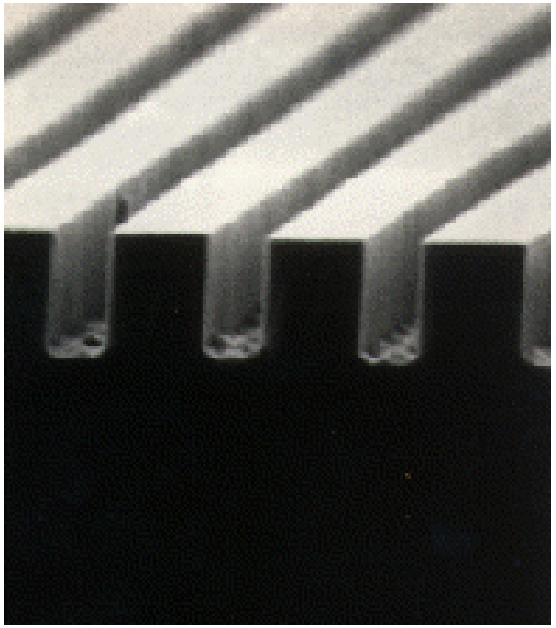


Diffraction gratings

The dots on a CD are equally spaced (although some are missing, of course), so it acts like a diffraction grating.

Any periodic surface is a diffraction grating

Gratings can work in reflection (r) or transmission (t).



Transmission gratings can be amplitude (α) or phase (n) gratings.

World’s largest diffraction grating

Lawrence Livermore National Lab

- Radiation
  - Generation of waves
  - Brewster's angle revisited

Where does light come from?

The wave equation describes the propagation of light.

But where does light come from in the first place?

Some matter must emit the light.

It does so through the matter's polarization:

$$\boxed{\vec{P}(t) = Nq\vec{x}_q(t)}$$

Note that matter's polarization is analogous to the polarization of light.

N: the number density of charged particles

q: is the charge of each particle

$\vec{x}_q(t)$ is the position of the charge.

Assuming each charge is identical and has identical motion.

How to take medium into account?

- In vacuum

$$\vec{D} = \varepsilon_0 \vec{E}$$

- Now that we have polarization

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

- If polarization is taken as scalar

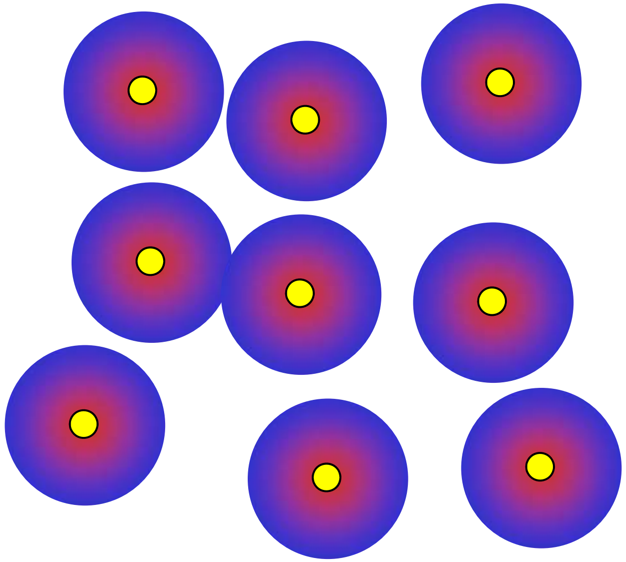
$$P = \varepsilon_0 (\chi^{(1)} E + h.o.t.)$$

$$\boxed{D = \varepsilon_0 (1 + \chi^{(1)}) E}$$

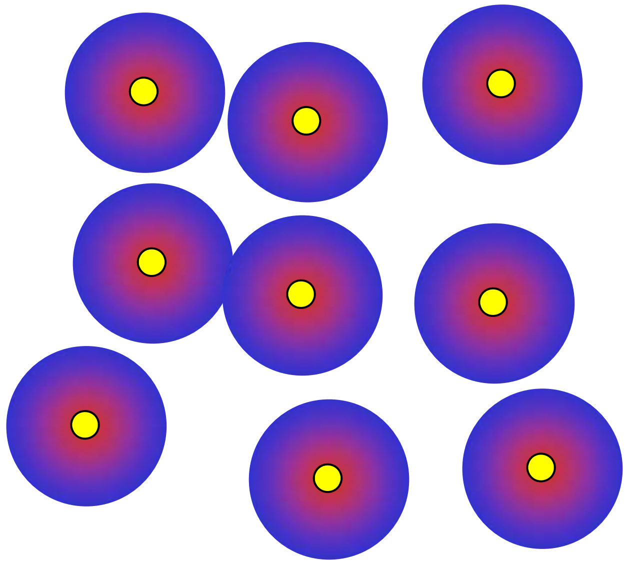
Polarized and unpolarized media

Unpolarized medium

(random phase)



Polarized medium



On the right, the displacements of the charges are correlated, so it is polarized at any given time.

Maxwell's equations for a medium

- The induced polarization, $\vec{P}$, contains the effect of the medium and is included in Maxwell's Equations:

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \frac{\partial \vec{P}}{\partial t}$$

This extra term turn it into the **Inhomogeneous Wave Equation**:

$$\frac{\partial^2 \vec{E}}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 N q \frac{\partial^2 \vec{x}_q}{\partial t^2}$$

The polarization is the **driving/source term** and tells us what light will be emitted.

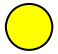
But $\partial^2 \vec{x}_q / \partial t^2$ is just the charge acceleration!

So it's accelerating charges that emit light!

Sources of light

Accelerating charges emit light

- **Linearly accelerating charge**

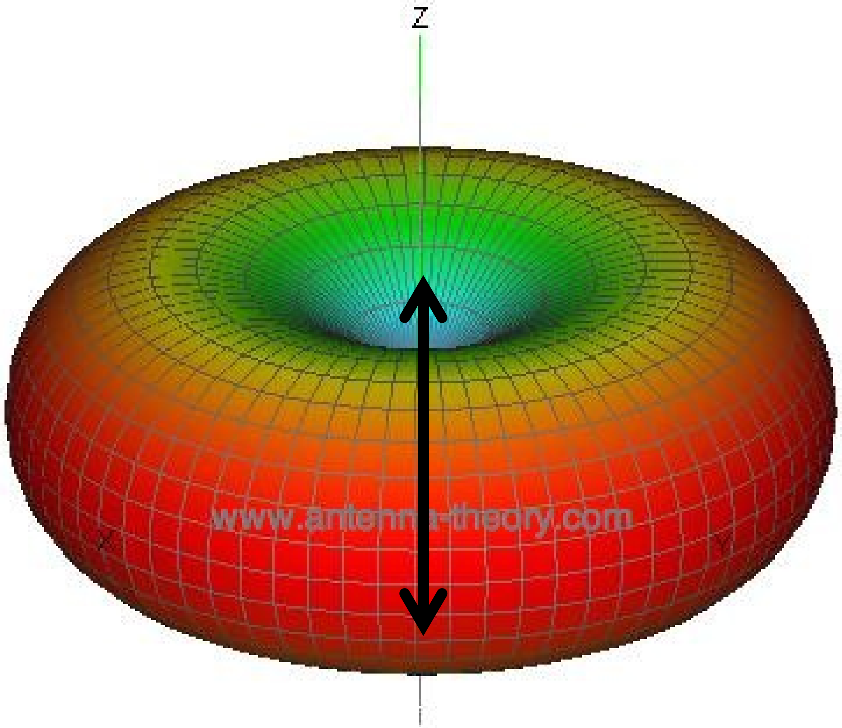


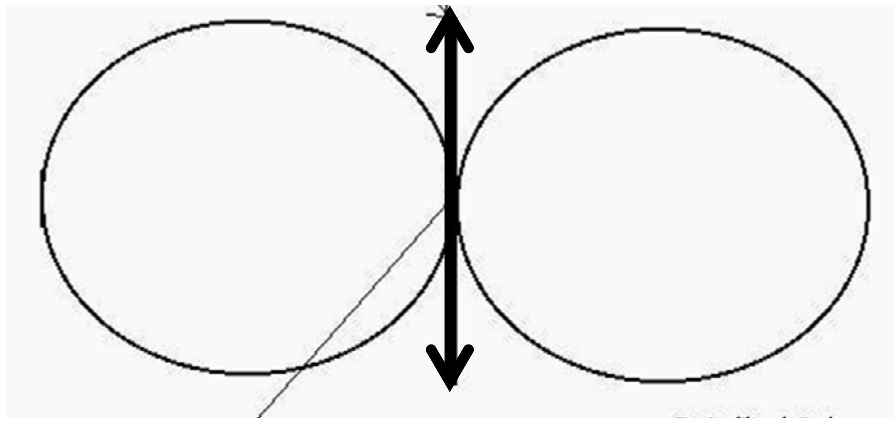
- Synchrotron radiation—
light emitted by charged
particles deflected by a
magnetic field

- Bremsstrahlung (Braking radiation)—
light emitted when charged particles
collide with other charged particles

Radiation pattern

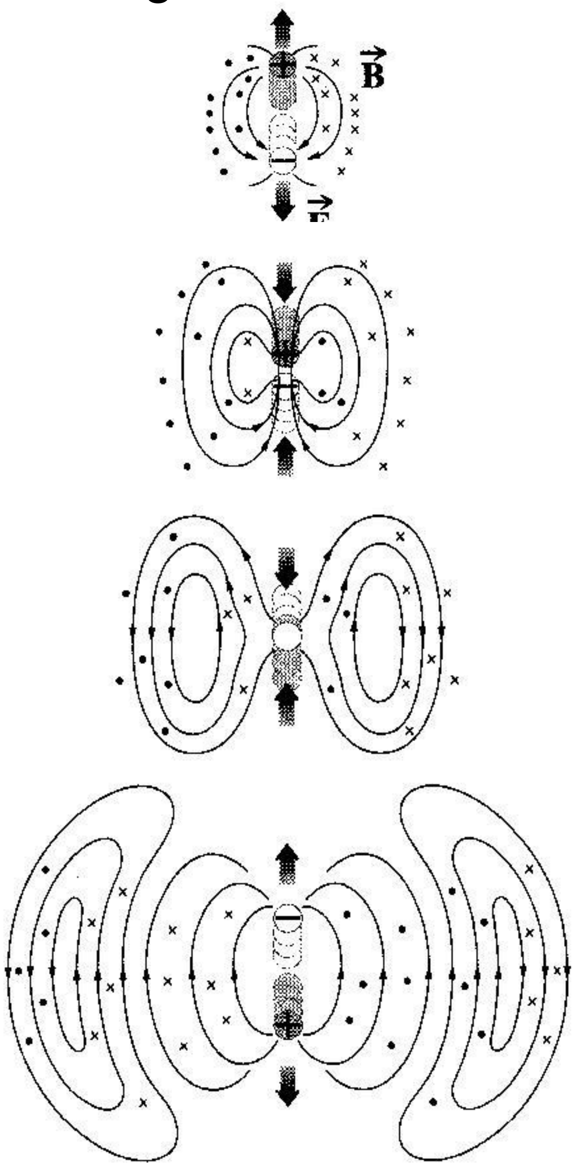
- Toroidal pattern
- No field in the direction of motion!!
  - How to align the antennas of your wireless router?

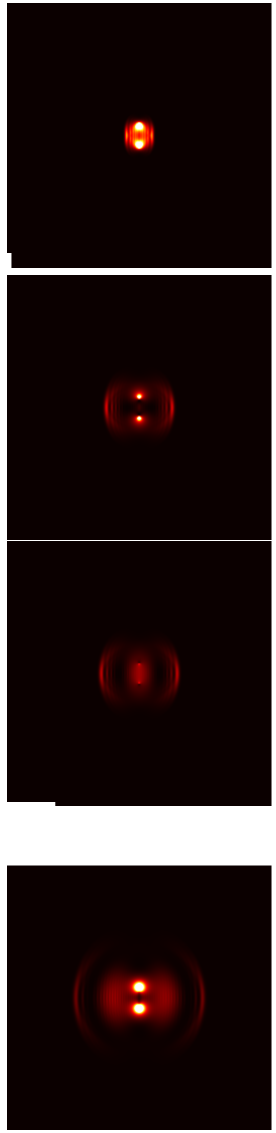




Electric dipole radiation

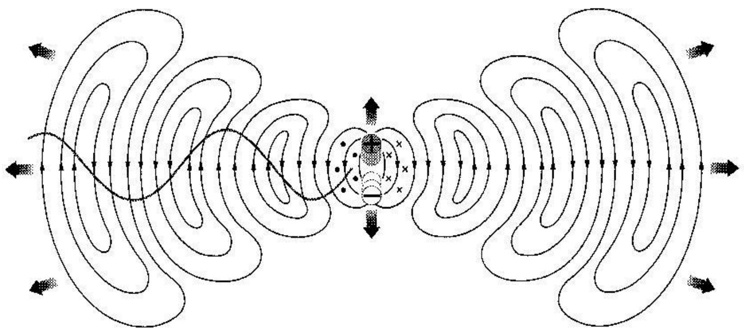
- Two spatially separated charges
  - One positive
  - One negative

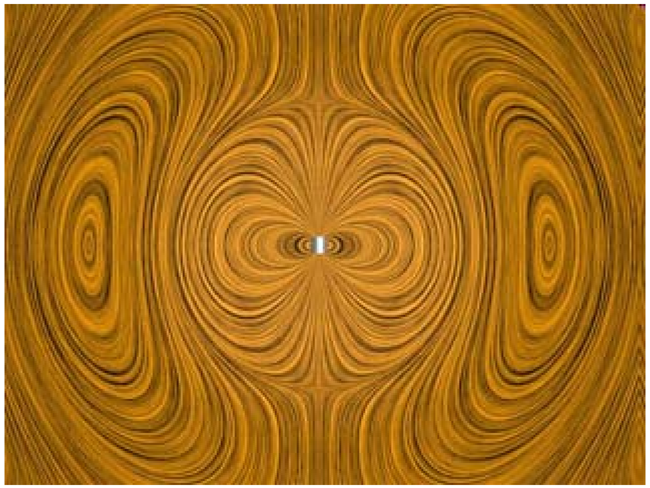


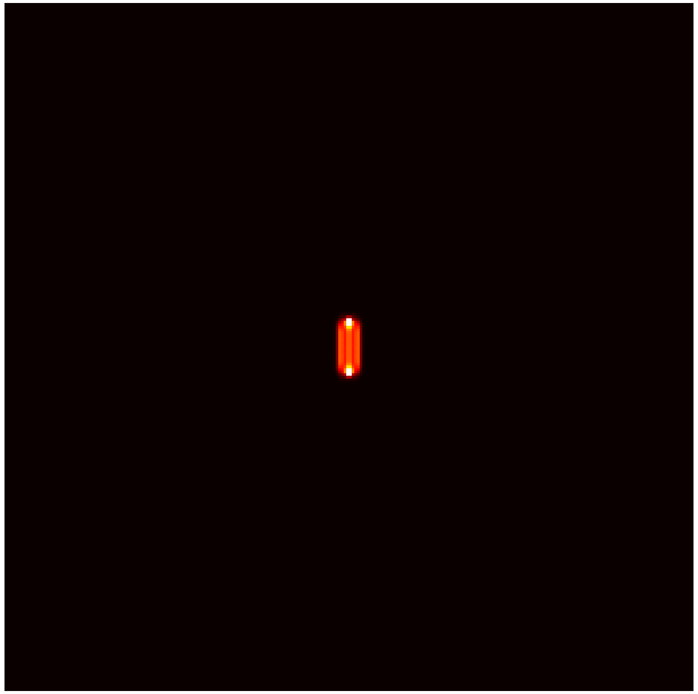


Electric dipole radiation-cont.

- Frequency, wavelength







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Brewster's angle revisited

- Why only TM mode?
- Recall that $\theta_B + \theta_t = \dfrac{\pi}{2}$

$$r_{TM} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_1 \cos\theta_t + n_2 \cos\theta_i} = 0$$

E_t // k_r !